

5. TRIANGLES

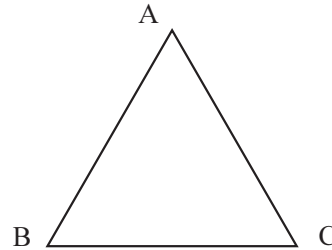
- * A simple closed figure formed by three line segment is called a Triangle
- * The line segmenets are called sides
- * Three non collineer points are formed a Triangle
- * A Triangle has 3 vertices, 3 sides and 3 angles.
- * A Trangledivides a plane inits 3 parts.
 - i) Interior of the Triangle
 - ii) Boundary of the Triangle
 - iii) Exterior of the Triangle
- * The boundary and Interior of the triangle together is called "Triangular region"

Euclid :

- * He is a famous Greek Philosopher and mathematician.
- * He has introduced geometry in a logical order in the book "The Elements"
- * His geometry known as Euclidion geometry.

Parts of Triangle

- i) Three sides AB or c.
 BC or a.
 AC or b.
- ii) Three vertices A, B, C
- iii) Three angles $\angle A$ (or) $\angle BAC$ (or) $\angle CAB$
 $\angle B$ (or) $\angle ABC$ (or) $\angle CBA$
 $\angle C$ (or) $\angle ACB$ (or) $\angle BCA$

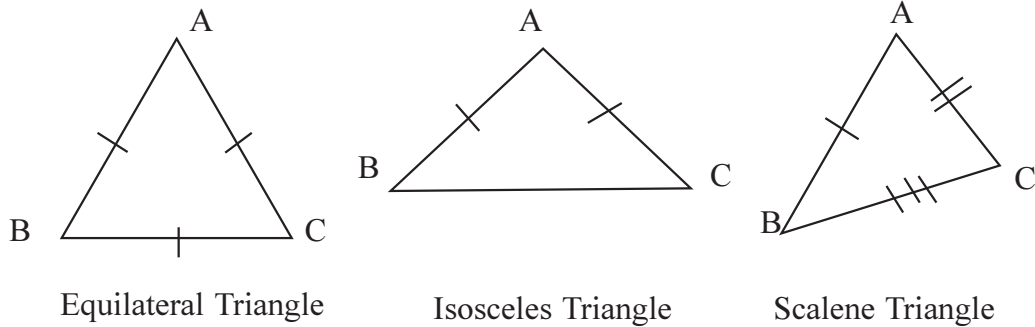


- * A triangle have 'Six elements' i.e. 3 sides and 3 angles.

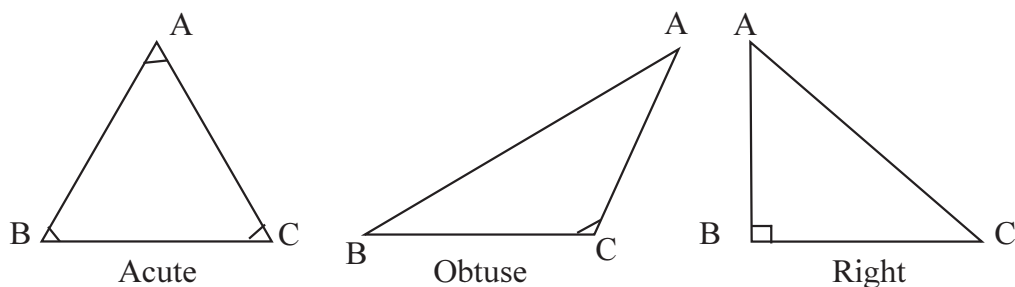
Classification of Triangles :

- * Based on the sides Triangles are 3 types.
 - * A Triangle having all 3 sides are equal length is called an Equilateral Triangle.

- * A Triangle having any two sides are equal length is called Isosceles Triangle.
- * If all the 3 sides of a Triangle are different length (or) Any two sides are not equal length is called a Scalene Triangle.



- * Based on the angles triangles are 3 types
 - * A triangle whose all angles are acute is called as acute angled triangle.
 - * A triangle whose one angle is obtuse is called an Obtuse angled triangle.
 - * A triangle whose one angle is right angle is called a right angled triangle.



- * There is only one obtuse angle is possible in a Triangle.
- * There is only one Right angle is possible in a Triangle.
- * In Equilateral Triangles the angles are 60° , 60° , 60° i.e. 1:1:1 Ratio.
- * In Right angle Triangle the side which is opposite to Right angle is called hypotenuse. In Right angle Triangle hypotenuse is the largest side.
- * In right Isosceles Triangle the angles are 45° , 90° , 45° i.e. 1:2:1 Ratio.
- * In a Right Isosceles Triangle the hypotenuse is $\sqrt{2}$ times to its side.
- * In a Right angle triangle the sums of other two angles is a right angle (or) other two angles are complementary angles.
- * In isosceles Triangle base angles are equal.
- * The sum of any two sides of a Triangle is greater than the third side.

- * The difference of any two sides of a triangle is less than the third side.
- * The perpendicular drawn from a vertex to the opposite side of a triangle is called an Altitude.
- * In acute angled triangle the altitude is interior of the triangle.
- * In obtuse angled triangle the altitude is exterior of the triangle.
- * In right angled triangle the altitude coincides with the side.
- * An altitude can be drawn from each vertex.
- * The line segment which is joining mid point of a side to the opposite vertex is called median.
- * The median lies in interior for every triangle.
- * The intersection of Median is called centroid (G)
- * The centroid divides each median in 2:1 ratio
- * The intersection of altitudes of a triangle is called ortho-centre (O)
- * The sum of three angles of a Triangle is 180° (or) Two right angles.
- * The Three angles of a Triangle form a straight angle.
- * An exterior angle of triangle is equal to the sum of its interior opposite angles.
- * The sum of Exterior angles of a Triangle is equal to 360° (or) 4 right angles (or) a complete angle.
- * To construct a Triangle we need three independent measures.
- * To construct an equilateral triangle we need one measure i.e. side.
- * To construct a Right angled Isosceles triangle we need one equal side.
- * It is not possible to construct a triangle when three angles are given.
- * Any two figures having same shape are called 'Similar' figures.
- * The symbol used to denote similar is '~'
- * **Similar Figures** : Two line segments, two circles, two squares, two equilateral triangles.
- * Any two figures having same shape and same size are called congruent figures.
- * The symbol used to denote congruence is ' $\cong$ ' i.e. '~' similar ; '=' Equal ..
- * If two line segments are said to be congruent they must have equal length.
- * If two circles are said to be congruent they must have equal radius.
- * If two figures are said to be congruent they must have equal side.

Congruency Properties :

- (1) Side - side - side congruency (S.S.S.) If three sides of a triangle are equal to corresponding three sides of another triangle then the two triangles are congruent..
 - (2) Side - Angle - Side Congruency (S.A.S.) If two sides and the angle between them is equal to the corresponding two sides and the angle between them, then the two triangles are congruent.
 - (3) Angle - Side - Angle Congruency : (A.S.A) If two angles and the included side of Triangle are equal to the two corresponding angles and included side of another triangle then the two triangles are congruent.
 - (4) Right angle - Hypotenuse - Side Congruency (R.H.S) If the Hypotenuse and one side of a right angled triangle are equal to the corresponding hypotenuse and side of the other right angled triangle then the two triangles are congruent.
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- * In two congruent triangles corresponding parts are equal.
 - * CPCT - Corresponding Parts of Congruent Triangles.
 - * Angles opposite to equal sides of an Isosceles Triangle are equal.
 - * The sides opposite to equal angles are equal.
 - * In any triangle the side opposite to larger angle is longer
 - * In congruent triangles there is a one- one correspondence between the vertices.
 - * In a Triangle the angles are 1:1:1 Ratio. Then opposite sides are also 1:1:1 Ratio.
 - * If the angles of triangle in 1:2:3 ratio then the opposite sides are in $1:\sqrt{3}:2$ Ratio.
 - * If the angles of a Triangle is 1:1:2 Ratio then the opposite sides are in $1:1:\sqrt{2}$ Ratio.
 - * The altitude of an equilateral triangle is $\frac{\sqrt{3}}{2}$ times to its side.
 - * The intersection of perpendicular bisectors of sides of a Triangle is called circumcentre.
 - * For acute angled triangle circum centre lies in its interior.
 - * For obtuse angled Triangle circumcentre lies in its exterior.

- * In Right angle triangle circum centre lies on hypotenuse and its mid point of Hypotenuase.
- * In Right angle triangle the radius of the circumcircle is half of the hypotenuse.
- * The circle which is passing through the vertices of the triangle is called circum circle.
- * For Scalene, Isosceles, equilateral Triangle the circum centre lies in its interior.
- * The intersection of Angular bisectors of a Triangle is called Incentre.
- * For any Triangle Incentre lies in its Interior.
- * In equilateral triangle circum centre (S) In centre (I), Ortho centre (O), Centroid (G) are coincide.
- * In Isosceles Triangle S, I, O, G are lies on the same line. They are called collinear points. The line is called 'Euler' line.
- * The circle which is passing through the sides of triangle is called incircle.
- * In Right angle triangle square of hypotenuse is equal to the sum of the square of other two sides
- * In similar Triangles
 - i) Corresponding sides are in proportion.
 - ii) Corresponding angle are equal.
- * In a triangle the line joining the mid points of any two sides is parallel to the third side and half in the third side.

SOLVED PROBLEMS

1. Find x and y from the adjacent figure

Sol : From the diagram $AB = AC \implies \angle ABC = \angle ACB = x^\circ$

$\angle ACB$ and $\angle ACD$ are formed a linear pair

$$\therefore \angle ACB + \angle ACD = 180^\circ$$

$$x + y = 180^\circ \dots\dots\dots (1)$$

from the diagram $\angle BAC = 80^\circ$ [Opposite vertical angles]

$$\therefore \angle ACD = \angle BAC + \angle ABC$$

[Exterior angle is equal to the sum of opposite interior angles]

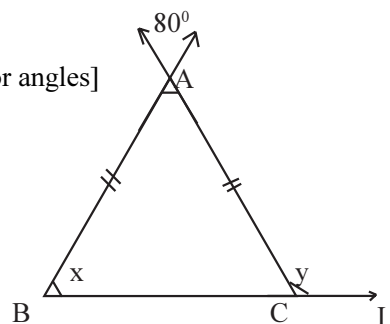
$$y = 80^\circ + x$$

$$y - x = 80^\circ \dots\dots\dots (2)$$

from (1) + (2)

$$x + y = 180^\circ; \quad x + 130 = 180^\circ; \quad x = 180 - 130$$

$$x = 50^\circ \quad y = 130^\circ$$



2. One of the exterior angle of a triangle is 125° and the interior opposite angles are in the ratio 2:3. Find the angles of the triangle ?

Sol : Exterior Angle = 125°

The Ratio of Interior angles = 2:3

Let the two interior angles = $2x$ & $3x$

$$\therefore 2x + 3x = 125^\circ$$

$$5x = 125$$

$$2x = 2 \times 25 = 50^\circ$$

$$x = \frac{125}{5} = 25^\circ$$

$$3x = 3 \times 25 = 75^\circ$$

$\therefore$ The angle of the triangle are $50^\circ, 75^\circ, 55^\circ$

3. Two poles of heights 9m and 14m stand upright on a plane ground. If the distance between their feet is 12 m. Find the distance between their Tops ?

Sol : From the adjacent figure

ΔAEC is a Rt. angled triangle

$AE = 12$; $CE = 5$; $AC = ?$

from Pythagoras Theorem

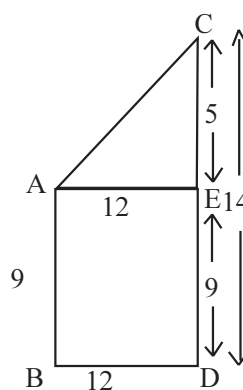
$$AC^2 = AE^2 + CE^2$$

$$AC^2 = 12^2 + 5^2$$

$$AC^2 = 144 + 25$$

$$AC^2 = 169$$

$$AC = \sqrt{169} = 13m$$



$\therefore$ the distance between their Tops is = 13m

4. In ΔABC , $AB = 5$ cm and $BC = 4$ cm find the range of values that CA can take?

Sol : We know $AB + BC > CA$ and $AB - BC < CA$

$$\implies CA < 9 \text{ cm and } CA > 1 \text{ cm}$$

$\therefore$ CA value lies between 1 cm and 9 cm

5. Can 6, 8, 11 be lengths of the sides of a right triangle ?

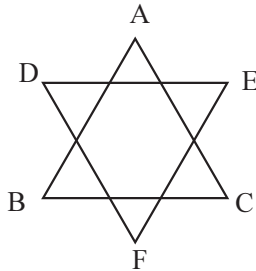
Sol : $11^2 = 121$

Sum of the squares of other two sides = $6^2 + 8^2 = 36 + 64 = 100$

Since $121 \neq 100$,

So 6, 8, 11 cannot be the lengths of the sides of a right triangle.

6. Find $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F$ from the adjacent figure



Sol : In $\triangle ABC$, $\angle A + \angle B + \angle C = 180^\circ$

In $\triangle DEF$, $\angle D + \angle E + \angle F = 180^\circ$

$\therefore \angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 180 + 180 = 360^\circ$

7. From the adjacent figure find x and y .

Sol : $\angle ABC + \angle CBX = 180^\circ$ (Linear pair)

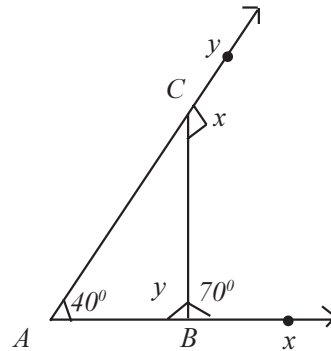
$70 + y = 180^\circ$

$y = 180 - 70 = 110^\circ$

In a triangle exterior angle is equal to sum of its opposite interior angles

$\angle BCY = \angle CAB + \angle ABC$

$x = 40 + y = 40 + 110 = 150^\circ$.



8. If D, E are points on sides AB, AC of $\triangle ABC$ such that $DE \parallel BC$.

If $\angle B = 30^\circ$ and $\angle A = 40^\circ$ find x, y, z

Sol : In $\triangle ABC$ we have

$\angle A = 40^\circ$ and $\angle B = 30^\circ$

$\angle A + \angle B + \angle C = 180^\circ$

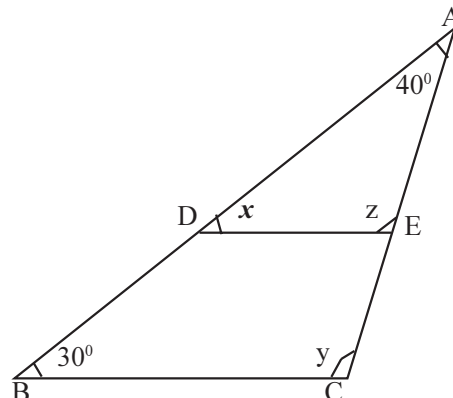
$\therefore \angle C = 110$

$y = 110$

Given $DE \parallel BC$

$z = y = 110^\circ$ (Corresponding Angles)

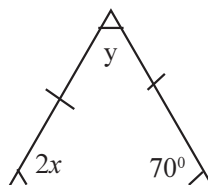
$x = 30^\circ$ (Corresponding Angles)



PRACTISE BITS :

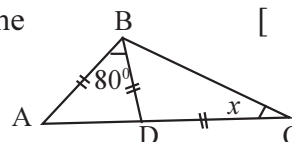
- If in $\triangle ABC$, $\angle A = 50^\circ$, $\angle B = 70^\circ$, $\angle C = 60^\circ$ then the ascending order of the sides is []
 a. $AB < BC < AC$ b. $BC < AB < AC$ c. $AC < AB < VC$ d. None
- If In a $\triangle ABC$, $\angle A = 60^\circ$ and $AB = AC$ then $\triangle ABC$ is []
 a. an isosceles triangle b. a right angled triangle
 c. an isosceles right angled triangle d. an equilateral triangle
- Two angles of a triangle are 40° and 63° the third angle must be []
 a. 67° b. 57° c. 87° d. 77°
- In which of the following cases a right angle triangle cannot be formed []
 a. 12 cm, 5 cm, 13 cm b. 8 cm, 6 cm, 10 cm
 c. 24 cm, 7 cm, 25 cm d. None of these
- In a $\triangle ABC$ If $AB^2 = BC^2 + AC^2$ then the right angle is at []
 a. A b. B c. C d. None
- If three angles of a triangle are in the ratio 2:3:10 then the largest angle []
 a. 24° b. 120° c. 36° d. 72°

- In the figure shows an isosceles triangle, the value of $x + y$ is



- a. 62° b. 70°
 c. 75° d. 83°

- Find the value of x given that ADC is a straight line

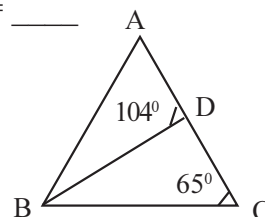


- a. 25° b. 30°
 c. 35° d. 40°

- If $AB = (2x + 20)cm$, $BC = \frac{3y}{5}cm$ and $AC = (3z - 20)cm$ where $x = 20cm$, $y = 100cm$, $z = 40cm$ then $\triangle ABC$

- a. an isosceles b. an equilateral c. a scalene d. none

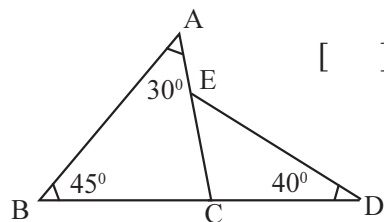
- In the figure $\angle BAD = 3\angle DBA$ then $\angle ABC = \underline{\hspace{2cm}}$



- a. 76° b. 19°
 c. 42° d. 58°

11. From the adjacent figure $\angle AED$ is equal to []

a. 75° b. 115°
c. 105° d. 100°



12. In a triangle ABC, the angles are $\angle A = 2x - 10^\circ$, $\angle B = x - 10^\circ$, $\angle C = x$ then which of the following is a right angle []

a. $\angle A$ b. $\angle B$ c. $\angle C$ d. None of these

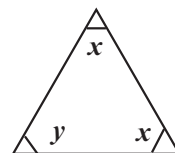
13. In triangle ABC and triangle XYZ, $AB = YZ$, $BC = ZX$, $AC = XY$ then which of the following is true []

a. $\angle A = \angle Z$ b. $\angle A = \angle X$ c. $\angle B = \angle X$ d. $\angle C = \angle X$

14. Find the relation between x and y in the adjacent figure expressed with x as subject []

a. $y = 180 + 2x$ b. $2x = 180 + y$

c. $x = \frac{1}{2}(180 - y)$ d. $x = \frac{1}{2}[180 + y]$



15. If two sides of an isosceles triangle are 3 cm and 8 cm then the length of the third side is []

a. 3 cm b. 8 cm c. 3 cm or 8 cm d. None of these

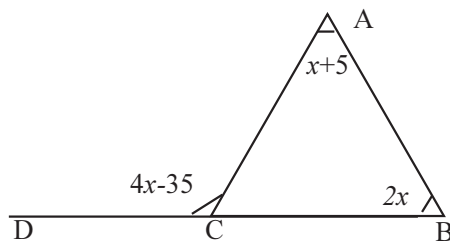
16. If in a $\triangle ABC$, $\angle A = 90^\circ$ and $AB = AC$ then $\triangle ABC$ is []

a. an Isosceles triangle b. a right angled triangle
c. an Isosceles right angled triangle d. an equilateral triangle

17. If $\angle A = x + 5^\circ$, $\angle B = 2x^\circ$ and

$$\angle ACD = 4x - 35^\circ$$

a. 45° b. 50°
c. 40° d. 35°



18. In a $\triangle ABC$, If $2\angle A = 3\angle B = 6\angle C$ then the measure of the smallest angle is

a. 90° b. 60° c. 40° d. 30°

19. In a $\triangle ABC$ If $\angle A + \angle B = 150^\circ$ and $\angle B + \angle C = 70^\circ$ then $\angle B$ []

a. 35° b. 45° c. 55° d. 25°

20. The lengths of two sides of a triangle are 6 cm and 9 cm then the possible length of the third side is []

a. 3 cm b. 15 cm c. 17 cm d. 10 cm

21. If the sides of a triangle are 4, 4, $4\sqrt{2}$ then the triangle is []

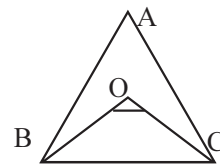
- a. Isosceles triangle b. Right angled triangle.
c. Right angled Isosceles triangle d. Equilateral triangle

22. One equal side of a Isosceles right angled triangle is $\sqrt{2}$ then its perimeter []

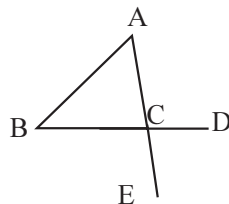
- a. $2(1+\sqrt{2})$ b. $4+\sqrt{2}$ c. $4+2\sqrt{2}$ d. $2+\sqrt{2}$

23. In $\triangle ABC$ the angle bisector of $\angle B$ and $\angle C$ meet at O. If $\angle A=60^\circ$ then find $\angle BOC$ []

- a. 60° b. 120°
c. 100° d. 80°



24. In the adjacent figure

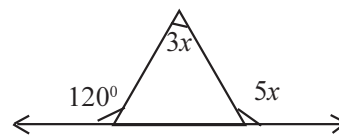


$\angle A=72^\circ$, $\angle B=50^\circ$, $\angle DCE=$

- a. 72° b. 50°
c. 58° d. 48°

25. From the adjacent figure value of x is []

- a. 20 b. 30
c. 40 d. 25



26. If the exterior angle of a triangle are $(2x+10)^\circ$, $(3x+5)^\circ$ and $(2x+40)^\circ$ then x []

- a. 45 b. 35 c. 25 d. 55

27. Which of the following is the set of measure of the sides of a triangle []

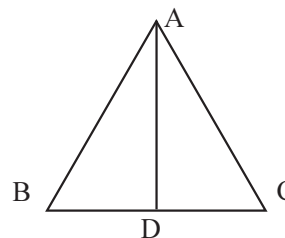
- a. 8, 4, 20 b. 9, 17, 25 c. 11, 16, 28 d. None of these

28. The point always lie in the interior of a triangle are []

- a. Orthocentre and Incentre b. Incentre and Circum centre
c. Centroid and Orthocentre d. Centroid and Incentre

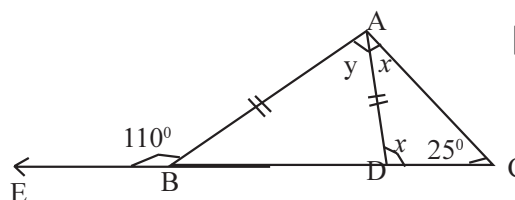
29. If $\angle BAD = \angle CAD$, $\angle ADB = \angle ADC$ then $\triangle ADB \cong \triangle ADC$ according to condition []

- a. SAS b. ASA
c. RHS d. SSS



30. A triangle has []
 a. Exactly two acute angles b. exactly one acute angle
 c. at least two acute angles d. exactly two obtuse angles
31. In a $\triangle ABC$ if $\angle A = \angle B + \angle C$ such that $\angle A = 3x$ then $x =$ []
 a. 30° b. 60° c. 90° d. 45°
32. In a $\triangle ABC$, which of the statement correct []
 a. $AB - BC > CA$ b. $AB + BC < CA$ c. $AB - BC < CA$ d. $AB + CA < BC$
33. Which of the following is equidistant from the vertices of the triangle []
 a. Incaentre b. Orthocentre c. Centroid d. Circumcentre
34. In $\triangle ABC$, $AB = AC$ and AD perpendicular to BC . The property b which $\triangle ADB$ is congruent to $\triangle ADC$ is []
 a. SAS b. SSS c. RHS d. AAA
35. If the sides of a triangle are doubled then its area []
 a. remains the same b. becomes doubled
 c. becomes three times d. becomes four times

36. In the given figure find $\angle Z$
 a. 40° b. 110°
 c. 45° d. 90°



137. Number of independent measurements required to constuct a triangle []
 a. 3 b. 4 c. 2 d. 1
38. In $\triangle ABC$ if $AB = AC$ and $\angle A = 50^\circ$ then []
 a. 60° b. 65° c. 70° d. 75°
39. If $\triangle ABC \cong \triangle PQR$ and $\angle B = x + 15^\circ$, $\angle Q = 70^\circ$, $\angle C = 5y + 10^\circ$ and $\angle R = 35^\circ$ by ASA congruence condition then the value of x and y is []
 a. $60^\circ, 5^\circ$ b. $55^\circ, 5^\circ$ c. $50^\circ, 5^\circ$ d. $45^\circ, 15^\circ$
40. If three angles of a triangle are in the ratio 1:2:3 then their opposite sides are in the ratio []
 a. 1:2:3 b. 1:1:2 c. $1:\sqrt{3}:2$ d. $1:\sqrt{2}:\sqrt{3}$
41. The three angles of a triangle are in the ratio 1:2:1 then the greatest angle is
 a. 45° b. 90° c. 60° d. 120°

42. In $\triangle ABC$ is an isoscales right angled at C such that AC=5cm then AB=

- a. 2.5 cm b. $5\sqrt{2}$ cm c. 10 cm d. 5 cm

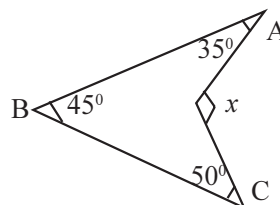
43. If the measures of the angles of a triangle are $(2x-5)^\circ$, $\left(3x-\frac{1}{2}\right)^\circ$ and

$\left(30-\frac{x}{2}\right)^\circ$ then x is []

- a. $\frac{311}{9}$ b. $\frac{309}{11}$ c. $\frac{310}{9}$ d. $\frac{301}{9}$

44. From the adjascent figure $x =$

- a. 130° b. 80°
c. 95° d. 85°



[]

45. In a right angled triangle if an angle measures 35° then the measure of other angle is []

- a. 65° b. 55° c. 45° d. 30°

* * *

TRIANGLES

KEY

1	B
2	D
3	D
4	D
5	C
6	B
7	C
8	A
9	A
10	D
11	C
12	A
13	D
14	C
15	B
16	C
17	C
18	D
19	B
20	D

21	C
22	A
23	B
24	C
25	B
26	A
27	D
28	D
29	B
30	C
31	A
32	C
33	D
34	C
35	D
36	C
37	A
38	B
39	B
40	C
41	B
42	B
43	A
44	A
45	B